University of California, Berkeley
Physics H7A Fall 1998 (*Strovink*)

SOLUTION TO EXAMINATION 1

1. In spherical polar coordinates, take Sapporo and Portland to be at $(r, \theta, \phi) = (R, \pi/4, 0) = \mathbf{r}_S$ and $(R, \pi/4, \pi/2) = \mathbf{r}_P$ respectively.

(a.) Here the course is one quarter of a circle with its center on the earth's axis of rotation at a point above the earth's center. This circle has radius $R \sin\theta = R\sqrt{2}/2$. The distance traveled, s, is one quarter of its circumference:

$$s = \frac{1}{4} 2\pi \frac{R\sqrt{2}}{2} = \frac{\pi R\sqrt{2}}{4} \ .$$

(b.) The Cartesian coordinates of $\mathbf{r}_S$ and $\mathbf{r}_P$ are

$$\mathbf{r}_S = R(\sqrt{2}/2, 0, \sqrt{2}/2)$$
$$\mathbf{r}_P = R(0, \sqrt{2}/2, \sqrt{2}/2) \ .$$

The angle between $\mathbf{r}_S$ and $\mathbf{r}_P$ is

$$\psi_{SP} = \frac{\arccos(\mathbf{r}_S \cdot \mathbf{r}_P)}{R^2} = \arccos(1/2) = \pi/3 \ .$$

To calculate the minimum distance between Sapporo and Portland along the surface of the earth, we bisect the earth using a plane that contains these two cities as well as the earth's center. The intersection of the earth with the bisecting plane is a circle with its origin at the center of the earth, having a circumference $2\pi R$. Since both $\mathbf{r}_S$ and $\mathbf{r}_P$ lie in this plane, the course consists of the fraction $\psi_{SP}/2\pi$ of this circumference. Therefore the distance traveled is

$$s = \frac{\psi_{SP}}{2\pi} 2\pi R = \frac{\pi/3}{2\pi} 2\pi R = \frac{\pi R}{3} \ .$$

(This is $0.333/0.354 \approx 94\%$ of the length of the "due east" course.)

2. Let $\theta = \arctan(5/12)$ be the angle with which the plane is inclined. Since there is no acceleration (or motion) perpendicular to that plane, the normal force N on block m must be equal to the normal component of the gravitational force on block m, or

$$N = mg\cos\theta \ .$$

The frictional force F_f then will be

$$F_f = \mu N = \mu mg\cos\theta$$

opposite to the motion.

(a.) Along the motion, the system consisting of mass m plus mass $m/13$ experiences a force due to gravity, consisting of the sum of the force $mg\sin\theta$ on mass m and $mg/13$ on mass $m/13$. If the velocity of the system is constant, *i.e.* there is no acceleration, the gravitational and frictional forces must balance:

$$\mu mg\cos\theta = mg(\sin\theta + 1/13)$$
$$\mu = \frac{5/13 + 1/13}{12/13} = 1/2 \ .$$

(b.) After the hanging block hits the table, the (massless) rope goes limp and has no further effect on the sliding block. *Opposite* to the direction of motion, the net force on m is

$$f = \mu mg\cos\theta - mg\sin\theta = mg\left(\frac{1}{2}\frac{12}{13} - \frac{5}{13}\right) = \frac{mg}{13} \ ,$$

producing a constant acceleration $a = g/13$ opposite to the motion. The block decelerates for a time $t = v_0/a$ until it comes to rest. During that time, the distance traveled is

$$s = v_0 t - \frac{1}{2}at^2 = \frac{v_0^2}{a} - \frac{1}{2}\frac{v_0^2}{a} = \frac{1}{2}\frac{v_0^2}{a} = \frac{13}{2}\frac{v_0^2}{g} \ .$$

3. Two crumpled paper objects have the same force (proportional to v^α) due to air resistance, but different masses $2m$ and m.

(a.) Immediately after the two objects are released from rest, their velocity must still be negligible; otherwise they would have experienced infinite acceleration. Likewise, the force of

air resistance, proportional to v^α, is negligible at that time. So the only nonnegligible force acting on them is the force of gravity, $2mg$ and mg respectively, yielding an acceleration g in either case. So the ratio of accelerations is $R = 1$.

(b.) After the objects reach terminal velocity (v_2 and v_1 respectively), and they no longer are accelerating, the forces due to air resistance and gravity must cancel:

$$\begin{aligned} 2mg &= Kv_2^\alpha \\ mg &= Kv_1^\alpha \ , \end{aligned}$$

where K is the unknown constant of proportionality. Taking the ratio of these two equations,

$$v_2 = 2^{1/\alpha} v_1 \ .$$

The "long time" after the objects are dropped is very large compared to the "very soon" time at which they reach terminal velocity. So, to an excellent approximation, the distance they drop during the "long time" is proportional to the terminal velocity. Since mass $2m$ drops a factor $\sqrt{2}$ further,

$$v_2 = \sqrt{2} v_1 \ .$$

Comparing this to the previous equation,

$$\alpha = 2 \ .$$

(c.) After gravity is turned off, the force of air resistance Kv^2 accelerates the falling object opposite to its direction of motion:

$$m\frac{dv}{dt} = -Kv^2 \ .$$

Dividing through by v^2, multiplying through by dt, and integrating,

$$\frac{1}{v} = \frac{K}{m}t + C$$

where C is a constant of integration that will be negligible with respect to Kt/m when t is sufficiently large. Therefore, asymptotically as $t \to \infty$, the downward velocity will be inversely proportional to t. Integrating this statement, the asymptotic distance traveled will increase as $\ln t$, which is arbitrarily large when t is arbitrarily large. Therefore the distance fallen will be *unbounded*. [This result is similar to that obtained in Problem Set 3, Problem 6 (K&K 2.35).]

4. Since the ice is horizontal and frictionless, it cannot exert any force on the barbell in the $\hat{\mathbf{x}}$ direction, either as the result of a contact force or a frictional force. Therefore the x coordinate x_{CM} of the barbell's center of mass, initially at rest at $x = 0$, must remain at $x = 0$. When mass m hits the ice and the barbell is horizontal,

$$\begin{aligned} 0 = X_{CM} &= \frac{Mx_M + m(x_M + h)}{M + m} \\ \Rightarrow \quad x_M &= -h\frac{m}{M + m} \ . \end{aligned}$$