University of California, Berkeley
Physics 110B, Fall 2008 (*Strovink*)
PROBLEM SET 6

28.
You are an indefagitable runner of rest mass m, whose feet generate a constant force F_0 in the x direction (as observed in the *laboratory*). This force causes your (relativistic) momentum to increase linearly with laboratory time t.
(a.)
At $t = 0$, when you are at rest at the origin, your feet begin to exert this force. Thereafter, show that $\sinh\eta$, where η is your rapidity, is proportional to t, and find the constant of proportionality.
(b.)
At $t = t_1$, a laser pulse is shot from the origin in the x direction. How much of a head start t_1 do you require in order for the laser pulse never to catch up with you?

29.
At $t = 0$, a particle of rest mass m and charge e is at rest at the origin. It accelerates under the influence of a uniform static electric field $\vec{E} = \hat{z}E_0$.
(a.)
For $t > 0$, show that the relativistic solution for $z(t)$ is given by

$$\begin{aligned} z(t) &= \int_0^t c\beta_z(t)\,dt \\ \beta_z(t) &= \tanh\eta(t) \\ \eta(t) &= \sinh^{-1}\frac{eE_0}{mc}t\ . \end{aligned}$$

(b.)
Suppose instead that, at $t = 0$, the particle has an initial momentum $\vec{p}(0) = \hat{x}p_\perp$. Show that the solution for $z(t)$ is the same as in (a.), except that m is replaced by m_{eff}, where

$$m_{\text{eff}} = \sqrt{m^2 + p_\perp^2/c^2}\ .$$

This says that, under the influence of a uniform electrostatic field, the longitudinal motion of a particle with nonzero transverse momentum is the same as that of a heavier particle with zero transverse momentum.
(c.)
Under the conditions of part (b.), for $t > 0$ does $x(t)$ increase linearly with t? Explain.

30.
At $t = 0$, a particle of rest mass m and charge e is at the origin, with initial momentum $\vec{p}(0) = \hat{x}p_\perp$. It accelerates under the influence of a uniform static magnetic field $\vec{B} = \hat{z}B_0$.
(a.)
For $t > 0$, show that the relativistic solution for $x(t)$ and $y(t)$ is given by

$$\begin{aligned} x(t) &= \int_0^t c\beta_x(t)\,dt \\ y(t) &= \int_0^t c\beta_y(t)\,dt \\ \gamma_\perp mc\beta_x(t) &= p_\perp\cos\omega_c t \\ \gamma_\perp mc\beta_y(t) &= p_\perp\sin\omega_c t \\ \omega_c &= -\frac{eB_0}{\gamma_\perp m} \\ \gamma_\perp m &= \sqrt{m^2 + p_\perp^2/c^2}\ . \end{aligned}$$

(b.)
Suppose instead that, at $t = 0$, the particle has an initial momentum $\vec{p}(0) = \hat{x}p_\perp + \hat{z}p_\parallel$. Show that the solution for $x(t)$ and $y(t)$ is the same as in (a.), except that $\gamma_\perp$ is replaced by γ, where

$$\gamma m = \sqrt{m^2 + p_\perp^2/c^2 + p_\parallel^2/c^2}\ .$$

(c.)
Show that the result of (b.) alternatively can be expressed as the result of (a.) with m replaced by

$$m_{\text{eff}} = \sqrt{m^2 + p_\parallel^2/c^2}\ .$$

This says that, under the influence of a uniform magnetostatic field, the transverse motion of a particle with nonzero longitudinal momentum is

the same as that of a heavier particle with zero longitudinal momentum.
(d.)
Under the conditions of part (b.), for $t > 0$ does $z(t)$ increase linearly with t? Explain.

31.
At $t = 0$ at the origin of a spherical polar coordinate system in the lab, a point particle of charge q has velocity βc directed along the $\hat{z}$ (north polar) axis. It has been moving with that constant velocity for a long time.
(a.)
Starting with the Coulomb field in the particle's rest frame, and using the rules for relativistic transformation of EM fields, show that the electric field observed in the lab at $t = 0$ and $\vec{r} = (r, \theta, \phi)$ is

$$4\pi\epsilon_0 \vec{E} = \hat{r}\frac{q}{r^2}\frac{\gamma}{(\gamma^2\cos^2\theta + \sin^2\theta)^{3/2}} \ ,$$

where as usual $\gamma = 1/\sqrt{1-\beta^2}$.
(b.)
Show that this result is equivalent to Griffiths Eq. (10.68).

32.
Griffiths Problem 10.9(b).

33.
The general expression for the electromagnetic fields arising from a point particle of charge q moving with velocity $\vec{\beta}c$ and acceleration $\dot{\vec{\beta}}c$ is

$$\vec{E} = \vec{E}_{\rm v} + \vec{E}_{\rm a} \ \text{ with}$$
$$\vec{E}_{\rm v} = \frac{q}{4\pi\epsilon_0}\left\{\frac{1}{\mathcal{R}^2}\frac{(\hat{\mathcal{R}} - \vec{\beta})(1-\beta^2)}{(1-\hat{\mathcal{R}}\cdot\vec{\beta})^3}\right\}_{\rm ret}$$
$$c\vec{B} = \left\{\hat{\mathcal{R}} \times \vec{E}\right\}_{\rm ret} \ ,$$

where $\vec{E}_{\rm v}$ is the *velocity field*, and the *acceleration field* $\vec{E}_{\rm a}$ is given in a later problem. Here $\vec{r}$ is a vector from the origin to the observer, $\vec{w}(t)$ is a vector from the origin to the particle, $\vec{\mathcal{R}} \equiv \vec{r} - \vec{w}$, and the subscript "ret" means that quantities are to be evaluated at time $t_{\rm ret} = t - \mathcal{R}/c$.

Assume that $\vec{\beta}$ lies in the z direction and is a constant, so that the acceleration field vanishes. As usual $\theta = \cos^{-1}\hat{z}\cdot\hat{r}$. Choose the origin of coordinates to be the position of the particle at $t = 0$. At that time, show that...
(a.)

$$-ct_{\rm ret} = \gamma\big(\gamma\beta z + \sqrt{(\gamma\beta z)^2 + r^2}\big) \ ;$$

(b.)

$$\mathcal{R}(1 - \hat{\mathcal{R}}\cdot\vec{\beta}) = r\sqrt{1 - \beta^2\sin^2\theta} \ ;$$

(c.)

$$\vec{r} = \mathcal{R}(\hat{\mathcal{R}} - \vec{\beta}) \ .$$

34.
Under the conditions of the previous problem, and using the tools developed there, show that $\vec{E}_{\rm v}$ is equivalent to Griffiths Eq. (10.68).