University of California, Berkeley
Physics 110B, Fall 2008 (*Strovink*)
PROBLEM SET 2

1.
Consider a uniform static magnetic field

$$\vec{B} = \hat{z}\, B_0 \ ,$$

where B_0 is a constant.
(**a.**)
Show that $\vec{B}$ can arise from the vector potential

$$\vec{A}_a = -B_0 y\, \hat{x} \ .$$

(**b.**)
Show that $\vec{B}$ can arise from the vector potential

$$\vec{A}_b = \tfrac{1}{2} B_0 s\, \hat{\phi}$$

(s and ϕ are cylindrical coordinates).
(**c.**)
By coordinate-system-independent vector analysis, show that $\vec{B}$ can arise from the vector potential

$$\vec{A} = \tfrac{1}{2} \vec{B} \times \vec{r}$$

(remember that $\vec{B}$ is constant).
(**d.**)
Referring to Griffiths' Eq. (10.7), find the gauge function λ that accomplishes the gauge transformation from $\vec{A}_a$ to $\vec{A}_b$.

2.
Griffiths Problem 10.3.

3.
Griffiths Problem 10.5.

4.
In free space with $\rho = 0$ and $\vec{J} = 0$, show that all four Maxwell equations can be obtained correctly if the scalar potential V is assumed to vanish, while the vector potential $\vec{A}$ satisfies

$$0 = \nabla \cdot \vec{A}$$
$$0 = \big(\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\big)\vec{A} \ .$$

5.
Griffiths Problem 10.7.

6.
Consider the Levi-Civita density $\epsilon_{ijk} \equiv 1$ ($ijk =$ even permutation of 123); $\equiv -1$ (odd permutation of 123); $\equiv 0$ (otherwise). It is found, for example, in the cross product

$$(\vec{a} \times \vec{b})_i = \epsilon_{ijk} a_j b_k \ .$$

Note that summation over the repeated indices j and k is implied; their domain is $1 \le j, k \le 3$.
(**a.**)
Show that

$$\epsilon_{ijk}\epsilon_{klm} = \delta_{il}\delta_{jm} - \delta_{im}\delta_{jl} \ ,$$

where δ is the Kronecker delta function (whose elements are those of the unit matrix).
(**b.**)
The determinant of a 3×3 matrix is given by

$$\det A \propto \epsilon_{ijk} A_{il} A_{jm} A_{kn} \epsilon_{lmn} \ .$$

By considering the number of nonzero terms on the RHS, and comparing it to the number of terms you would have expected for a 3×3 determinant, deduce the constant of proportionality. Express it in terms of a factorial.
(**c.**)
Guessing the explicit constant of proportionality, write a similar equation for the determinant of a 4×4 matrix. How should ϵ_{ijkl} be defined?