University of California, Berkeley
Physics 110B, Fall 2008 (*Strovink*)
PROBLEM SET 12

71. Fuzzy thick slit.
Please use the notation and results of Problem 70. Consider a trapezoidal aperture function

$$\begin{aligned} g(x) &= 1 \qquad |x| < \tfrac{a}{2} \\ &= 0 \qquad |x| > a \\ &= \tfrac{2}{a}(x+a) \qquad -a < x < -\tfrac{a}{2} \\ &= \tfrac{2}{a}(a-x) \qquad \tfrac{a}{2} < x < a\ . \end{aligned}$$

Fraunhofer conditions apply. Under these conditions, calculate the slit's diffraction pattern

$$\mathcal{R}(\psi_x) \equiv \frac{I(\psi_x)}{I(\psi_x = 0)}\ .$$

72. Thick slits with wave plates.
A linearly ($\hat{x}$) polarized plane EM wave traveling along $\hat{z}$ is incident on an opaque baffle located in the plane $z = 0$. The baffle has two slits cut in it, which are of infinite extent in the $\hat{y}$ direction. In the $\hat{x}$ direction, the slit widths are each a and their center-to-center distance is d. (Obviously $d > a$, but you may *not* assume that $d \gg a$.) The top and bottom slits are each an equal distance from $x = 0$.

The diffracted image is viewed on a screen located in the plane $z = L$, where $L \gg d$; also $\lambda L \gg d^2$, where λ is the EM wavelength.

Quarter-wave plates are placed in each slit. They are identical, except that the top plate's "slow" (high-index) axis is along $(\hat{x}+\hat{y})/\sqrt{2}$ ($+45°$ with respect to the $\hat{x}$ axis), while the bottom plate's slow axis is along $(\hat{x}-\hat{y})/\sqrt{2}$ ($-45°$ with respect to the $\hat{x}$ axis).
(a.)
What is the state of polarization of the diffracted light that hits the center of the screen, at $x = y = 0$? Explain.
(b.)
At what diffracted angle ψ_x does the first minimum of the irradiance occur?

73.
Consider a piece of plate glass with refractive index $n = \sqrt{2}$ and no magnetic properties, *i.e.* $\mu = \mu_0$. The glass is $\sim$1 cm thick, but, since this is a white light problem, its exact thickness doesn't matter. You may take the region outside the glass to have $n = 1$.

A beam of white light from a distant flashlight is normally incident on the glass; the beam diameter is d. You view the transmitted light on the axis of this beam, at a distance D far enough downstream that $D\lambda \gg d^2$.
(a.)
When the glass is inserted into the flashlight beam, you see its irradiance diminish to a fraction F of its former value. What is F?
(b.)
Now you paint one surface of the glass, using fast-drying black paint from a spray can. But the nozzle isn't working properly, and you hold it too far away from the glass. Instead of a smooth coat, the paint forms a rough surface that is not fully opaque. Through a magnifying glass, you see many little blobs of paint stuck to the glass. The blobs themselves are opaque, much larger than a typical wavelength, and uniformly distributed – but half of the glass area isn't covered by a blob.

In terms of F, what new fraction F' of the original (no-glass) irradiance do you observe?

74.
Here's a practical optics problem using real numbers. A small white LED is placed 1 m upstream of a 1 cm diameter circular hole in an otherwise opaque baffle. The hole is uniformly illuminated by the LED. The diffracted irradiance is observed infinitely far downstream (the observation distance can be made finite by introducing a field lens, but let's omit the lens here). The light source, hole center, and observer are lined up on the z axis. Using this setup, the observer sees a finite irradiance I_0.

In the following, you are asked to consider the new irradiance I_x after the setup is modified as

described. Each modification is independent – for example, modification (b.) does not include modification (a.).

(a.)

The circular hole is now fully blocked, except that a pie-shaped opening allows its first quadrant to remain clear. Calculate I_a/I_0.

(b.)

A monochromatic source of visible light (for example a thin laser beam that is caused to diverge by an adjacent strong concave lens) is substituted for the LED. The (full) hole still is illuminated uniformly, and the total light power entering the hole is the same as for the LED. Can you claim that I_b would be approximately the same as I_0? Explain.

(c.)

Here the white LED is restored as the light source, but the hole diameter is reduced from 1 cm to 0.5 cm. Can you claim that I_c would be approximately the same as I_a? Explain.

75.

A Survivor contestant tries to signal a blimp hovering nearly overhead. It is pitch dark, and his only source of light is an infinitesimal, monochromatic, isotropic-light-emitting diode (LED). The naked LED isn't quite bright enough to be seen by his blimp-borne rescuer. Remembering Physics 110B, the contestant resolves to amplify the light signal that the rescuer perceives.

(a.)

The contestant stretches a large opaque plastic sheet over a flat frame and pokes a small (couple of mm dia) circular hole in it. He carefully positions the hole directly between the LED and the blimp, separated from the LED by a couple of meters. Relative to the naked LED, is it possible that the irradiance seen by the rescuer increases? If so, by what maximum factor?

(b.)

Lacking a plastic sheet, the contestant disassembles his bicycle hub to obtain a small (couple of mm dia) blackened steel ball. Using a spiderweb thread, he carefully hangs the ball directly between the LED and the blimp, separated from the LED by a couple of meters. Relative to the naked LED, is it possible that the irradiance seen by the rescuer increases? If so, by what maximum factor?