University of California, Berkeley
Physics 110B, Fall 2008 (*Strovink*)
PROBLEM SET 1

-6.
(**a.**)
Taking $\hat{z}$ to be its direction of propagation, a plane monochromatic electromagnetic wave with angular frequency ω is described by the electric and magnetic fields

$$\vec{E} = \mathrm{Re}\Big(\tilde{\mathbf{E}} \exp\big(i(\tilde{k}z - \omega t)\big)\Big)$$
$$\vec{H} = \mathrm{Re}\Big(\tilde{\mathbf{H}} \exp\big(i(\tilde{k}z - \omega t)\big)\Big) ,$$

where $\tilde{\mathbf{E}}$ and $\tilde{\mathbf{H}}$ are complex constant vectors, and $\tilde{k}$ is a constant scalar, complex if the medium is conducting.

In an *insulating* medium with fixed uniform dielectric constant ϵ and magnetic permeability μ, $\tilde{k}$ is real. Show that Maxwell's equations reduce to

$$\tilde{E}_z = 0$$
$$\tilde{H}_z = 0$$
$$k\hat{z} \times \tilde{\mathbf{E}} = \mu\omega\tilde{\mathbf{H}}$$
$$k\hat{z} \times \tilde{\mathbf{H}} = -\epsilon\omega\tilde{\mathbf{E}} .$$

-5.
(**a.**)
Under the conditions of Problem **-6**, how many different real constants must be supplied in order to fully specify *e.g.* $\tilde{\mathbf{E}}$? Once $\tilde{\mathbf{E}}$ is specified, how many more additional real constants are needed to specify $\tilde{\mathbf{H}}$?
(**b.**)
Under these same conditions, show that the time-averaged energy flux density of the electromagnetic wave is

$$\langle \vec{S} \rangle = \tfrac{1}{2}\mathrm{Re}(\tilde{\mathbf{E}} \times \tilde{\mathbf{H}}^*) .$$

Would this be true in a *conducting* medium?

-4.
Consider a plane EM wave in medium 1 that is normally incident on a plane interface with medium 2. In lecture we derived the amplitude reflection coefficient

$$\tilde{R} = \frac{\tilde{Z}_1^{-1} - \tilde{Z}_2^{-1}}{\tilde{Z}_1^{-1} + \tilde{Z}_2^{-1}} ,$$

where, using the notation of Problem **-6**, the *inverse characteristic impedance* of the medium is

$$\tilde{Z}^{-1} \equiv \frac{\tilde{H}}{\tilde{E}} ,$$

and $\vec{E}$ is assumed to lie along $\hat{x}$ and $\vec{H}$ along $\hat{y}$.
(**a.**)
In a *conducting* medium with fixed uniform magnetic permeability μ, show that Faraday's law still requires, as in Problem **-6**,

$$\tilde{k}\tilde{E} = \mu\omega\tilde{H}$$
$$\Rightarrow \tilde{Z}^{-1} = \frac{\tilde{k}}{\mu\omega} .$$

(**b.**)
Using the full Maxwell equations and allowing for nonzero constant conductivity σ (but neglecting any volume free charge density), we showed in lecture that

$$\frac{\tilde{k}}{\omega/v} \equiv r + is \text{ , where}$$
$$r = \sqrt{\frac{\sqrt{1+\gamma^2}+1}{2}}$$
$$s = \sqrt{\frac{\sqrt{1+\gamma^2}-1}{2}}$$
$$\gamma \equiv \frac{\sigma}{\epsilon\omega}$$
$$v \equiv \sqrt{\frac{1}{\epsilon\mu}} .$$

Considering the special case in which medium 1 is an insulator and medium 2 is a conductor, solve for the reflectivity $\tilde{R}$ and show that your result is consistent with Griffiths Eq. 9.147(a).

-3.
(**a.**)
Using the results of Problem **-4**, and considering the limiting case of a poor conductor ($\gamma \ll 1$), show that both $\vec{E}$ and $\vec{H}$ are attenuated by a factor e for every distance

$$\Delta z = \frac{2}{\sigma}\sqrt{\frac{\epsilon}{\mu}}$$

that is traversed by the plane wave.
(**b.**)
Consider again the problem of reflection at normal incidence. Suppose that both media have the same ϵ/μ, but medium 1 is an insulator while medium 2 has a small but nonzero conductivity. In the poor-conductivity limit show that

$$\tilde{R} = -\frac{i\gamma}{4} \ .$$

-2.
Seawater has a resistivity of ≈ 0.2 ohm-meters, and the dielectric constant of seawater is $\epsilon/\epsilon_0 \approx 80$.
(**a.**)
Using reasonable approximations, estimate the angular frequency ω at which the amplitude of electromagnetic waves in seawater will diminish by a factor e for every 10 meters traversed.
(**b.**)
A dial-up modem allows communication at a maximum of 56 kilobits per second over telephone lines working at audio frequencies ($\omega \leq\approx 2\pi \times 10^4$ Hz). If electromagnetic waves exchanged between satellites and submarines must penetrate ≈ 30 m of seawater without being attenuated by more than a factor ≈ 400 in power, estimate the maximum number of bits per second that can be exchanged.

-1.
You shine a 1W flashlight on a crow, who absorbs its beam. (Model the mechanical and thermal properties of a crow as those of 100 g of water.)
(**a.**)
How long must the flashlight shine to heat the crow by 1 degree Celsius?
(**b.**)
How long must the flashlight shine to accelerate the crow from rest to a velocity of 1 mm/sec (assuming no other forces are acting)?

0.
Griffiths Problem 9.18.