University of California, Berkeley
Physics 105 Fall 2006 Section 2 (*Strovink*)

ASSIGNMENT 7

Reading:
SCCM 6.1-6.4
Taylor 13.1-13.7

1. Taylor Problem 13.14.

2. Taylor Problem 13.17.

3. Taylor Problem 13.23.

4. Taylor Problem 13.25.

5.
The interaction Lagrangian for a system consisting of a relativistic test particle of mass m and charge e moving in a static electromagnetic field is

$$\mathcal{L}(\mathbf{x},\mathbf{v},t) = -mc^2\sqrt{1-\frac{v^2}{c^2}} + e\mathbf{v}\cdot\mathbf{A} - e\phi \ ,$$

where $\mathbf{x}$ is the particle's position, $\mathbf{v}$ is its velocity, $\phi(\mathbf{x})$ is the electrostatic potential ($\mathbf{E} = -\nabla\phi$), $\mathbf{A}$ is the (static) magnetic vector potential ($\mathbf{B} = \nabla\times\mathbf{A}$), and c is the speed of light.
(**a**)
Write down the canonical momenta (p_1, p_2, p_3) which are conjugate to the Cartesian coordinates (x_1, x_2, x_3).
(**b**)
Compute the Hamiltonian $\mathcal{H}(\mathbf{x},\mathbf{v},t)$.
(**c**)
Re-express $\mathcal{H}(\mathbf{x},\mathbf{v},t)$ as the function $\mathcal{H}(\mathbf{x},\mathbf{p},t)$.
(**d**)
Show that $\mathcal{H}$ is conserved. Is it equal to

$$E = \frac{mc^2}{\sqrt{1-\frac{v^2}{c^2}}} \ ,$$

the total (relativistic) energy of the test particle? Explain.

6.
Consider f and g to be any two continuous functions of the generalized coordinates q_i and canonically conjugate momenta p_i, as well as time:

$$f = f(q_i, p_i, t)$$
$$g = g(q_i, p_i, t) \ .$$

The *Poisson bracket* of f and g is defined by

$$[f,g] \equiv \frac{\partial f}{\partial q_i}\frac{\partial g}{\partial p_i} - \frac{\partial f}{\partial p_i}\frac{\partial g}{\partial q_i} \ ,$$

where summation over i is implied. Prove the following properties of the Poisson bracket:
(**a**)

$$\frac{df}{dt} = [f,\mathcal{H}] + \frac{\partial f}{\partial t}$$

(**b**)

$$\dot{q}_i = [q_i,\mathcal{H}]$$

(**c**)

$$\dot{p}_i = [p_i,\mathcal{H}]$$

(**d**)

$$[p_i,p_j] = 0$$

(**e**)

$$[q_i,q_j] = 0$$

(**f**)

$$[q_i,p_j] = \delta_{ij} \ ,$$

where $\mathcal{H}$ is the Hamiltonian. If the Poisson bracket of two quantities is equal to unity, the quantities are said to be *canonically conjugate*. On the other hand, if the Poisson bracket vanishes, the quantities are said to *commute*.
(**g**)
Show that any quantity that does not depend explicitly on the time and that commutes with the Hamiltonian is a constant of the motion.

7.
Taylor Problem 13.35. What is the physical meaning of this problem's result? (Consider the angular divergence of the beam.)