University of California, Berkeley
Physics 105 Fall 2006 Section 2 (*Strovink*)

ASSIGNMENT 10

Reading:
SCCM 12.1-12.6, 13.1-13.3, 14.1-14.3
Taylor 11.1-11.7, 16.1-16.3

1. Considering once more the shock-absorberless El Dorado in Wim Wenders' & Sam Shepard's *Paris, Texas* (Cannes 1984 Palme d'Or), continue to model it as a thin horizontal rectangular plate of mass m, length l and width w, supported by four identical springs of constant k at the corners. Let's make a clairvoyant choice of general coordinates: q_1 is the CM height, q_2 is the angle of CCW rotation about the short bisector, and q_3 is the angle of CCW rotation about the long bisector. All three coordinates vanish at equilibrium, with $|q_2|$ and $|q_3|$ both $\ll 1$.

(**a.**)
In terms of the q_i, write down the $\mathcal{M}$ and $\mathcal{K}$ matrices. From their form, show that the chosen general coordinates are proportional to the normal coordinates.

(**b.**)
Find the natural frequencies of oscillation and show that two of them are degenerate.

(**c.**)
A pair of potholes set the Eldo into oscillatory rotation about an *arbitrary* horizontal axis passing through its CM. Discuss its ensuing motion.

2. As a warmup for the double pendulum problems that follow, use Lagrangian methods to obtain the equation of motion for a single *driven upright pendulum*. It consists of a bob (mass m) supported by a massless stick (length h) pivoted on a support that moves vertically according to $Y(t) = Y_0 \cos \Omega t$. The stick makes a CCW angle ψ with the vertical direction, measured from the top: when $\psi = 0$, the bob is *upright*.

(**a.**)
Using ψ and Y as generalized coordinates, write the Lagrangian.

(**b.**)
Write the Euler-Lagrange equation in ψ, then plug in $Y(t) = Y_0 \cos \Omega t$.

(**c.**)
Beautify this equation by introducing the parameters $a \equiv (2\omega_0/\Omega)^2$ and $q \equiv 2Y_0/h$, where $\omega_0^2 \equiv g/h$. Also introduce the "scaled time" $\tau = \Omega t/2$. You should obtain the *Mathieu equation*

$$\frac{\mathrm{d}^2\psi}{\mathrm{d}\tau^2} - (a - 2q\cos 2\tau)\sin\psi = 0 \ .$$

When we discuss parametric resonance in lecture, we will show that the upright pendulum is stable if $\Omega \gg \omega_0$ and $q^2 > 2a$, that is if the support vibrates very fast with amplitude no smaller than a fraction $\sqrt{2}\omega_0/\Omega$ of h.

3. Taylor Problem 11.15.

4. Taylor Problem 11.16.

5. Taylor Problem 11.19.

6. Taylor Problem 16.9.

7. A long narrow rectangular membrane, when undisturbed, lies in the x-y plane; $\hat{x}$ is its long direction and $\hat{y}$ is its short direction. The membrane's displacement normal to the x-y plane is denoted by $z(x, y, t)$. The membrane is clamped at its long edges $y = 0$ and $y = b$, so that

$$z(x, 0, t) = z(x, b, t) = 0 \ .$$

Consider the propagation of traveling sinusoidal waves $z(x, y, t)$ in the long direction $\hat{x}$.

(**a.**)
The Lagrangian density $\mathcal{L}'$ (per unit area of

membrane) is given by

$$\mathcal{L}'(z, \tfrac{\partial z}{\partial x}, \tfrac{\partial z}{\partial y}, \tfrac{\partial z}{\partial t}, x, y, t) =$$
$$= \tfrac{1}{2}\sigma\left(\tfrac{\partial z}{\partial t}\right)^2 - \tfrac{1}{2}\beta\left(\left(\tfrac{\partial z}{\partial x}\right)^2 + \left(\tfrac{\partial z}{\partial y}\right)^2\right) ,$$

where σ is the membrane's mass per unit area, and β is a positive constant that is inversely proportional to its elasticity. Apply the Euler-Lagange equations to this Lagrangian density to obtain a partial differential equation for $z(x, y, t)$.

(**b.**)
Search for a trial solution in the form

$$z(x, y, t) = Y(y) \cos(k_x x - \omega t) ,$$

where $Y(y)$ is a function only of y, and k_x and ω are constants that are not yet specified. Plug this solution into the equation you obtained for (**a.**). Dividing through by $\cos(k_x x - \omega t)$, obtain an ordinary differential equation for $Y(y)$.

(**c.**)
Applying the boundary conditions $z(x, 0, t) = z(x, b, t) = 0$, identify and choose a (non null) solution for $Y(y)$ which has the most gradual dependence on y that is possible given these conditions.

(**d.**)
Returning to the equation you obtained for (**a.**), plug in your answer to (**c.**) to obtain an equation relating k_x and ω.

(**e.**)
What is the minimum frequency $\omega_{\min}$ of sinusoidal waves that can propagate in the $\hat{x}$ direction without attenuation?

(**f.**)
If $\omega = \sqrt{2}\,\omega_{\min}$, calculate the phase velocity ω/k_x with which sinusoidal waves propagate in the $\hat{x}$ direction.