

**ASSIGNMENT 1**

**Reading:**

SCCM 1.1-1.5.

Taylor 1.7, p. 623, 4.8, Example 15.6, 9.3-4, pp. 401-402.

**1.** Taylor Problem 1.23. *Hint:* Because  $\mathbf{c} = \mathbf{b} \times \mathbf{v}$  is  $\perp$  to  $\mathbf{v}$ ,  $\mathbf{v}$  lies in the plane defined by two different vectors that are both  $\perp$  to  $\mathbf{c}$ . Take these two vectors to be  $\mathbf{b}$  and  $\mathbf{b} \times \mathbf{c}$ . Write  $\mathbf{v}$  as a linear sum of them and solve for the linear coefficients.

**2.**

(a.)

Prove for any vector  $\mathbf{b}$  and index  $i$  that

$$\epsilon_{ijk} b_j b_k = 0 ,$$

where  $\epsilon_{ijk}$  is the Levi-Civita density, and summation over repeated indices is implied.

(b.)

Prove for any  $i, j, l, m$  that

$$\epsilon_{ijk} \epsilon_{klm} = \delta_{il} \delta_{jm} - \delta_{im} \delta_{jl} ,$$

where  $\delta_{ij}$  is the Kronecker delta function.

**3.** Work Taylor Problem 1.23 again, as before starting from

$$\mathbf{v} = \alpha \mathbf{b} + \beta \mathbf{b} \times \mathbf{c} ,$$

but this time writing all vector products in component form using the Levi-Civita density. Do not use any vector relations or identities. The results of Problem 2 will help.

**4.** Taylor Problem 1.47.

**5.** Taylor Problem 1.48.

**6.**

(a.)

Deduce from first principles the general form of a real orthogonal  $2 \times 2$  matrix.

(b.)

Supposes this  $2 \times 2$  matrix is a submatrix of a  $3 \times 3$  matrix, whose third row and third column vanish except for the (3,3) element which is unity. What restrictions are placed upon the form of the  $2 \times 2$  matrix if the  $3 \times 3$  matrix corresponds to a proper rotation? To an improper (parity-inverting) rotation? Explain.

**7.** Taylor Problem 10.48.