University of California, Berkeley
Physics 105 Sec. 2 Fall 2006 (*Strovink*)

FINAL EXAMINATION

Directions: Do all six problems, which have unequal weight. This is a closed-book closed-note exam except for *Short Course in Classical Mechanics*, Taylor, a copy of anything posted on the course web site, and anything in your own handwriting (not a Xerox of someone else's writing). Laptops, palmtops, calculators, and cellphones should be turned off. Use a bluebook. Do not use scratch paper – otherwise you risk losing part credit. Show all your work. Cross out rather than erase any work that you wish the grader to ignore. Justify what you do. Express your answer in terms of the quantities specified in the problem. Box or circle your answer.

1. (35 points)
In the body axes, a cube of mass m and side $2a$ is centered at the origin and has faces normal to the x, y, and z axes, respectively. About an axis that points from corner $(-a, -a, -a)$ to corner (a, a, a), at $t = 0$ the cube is rotating with angular velocity ω_0.

(**a.**) (15 points)
In the body system at $t = 0$, find the angular momentum $\vec{L}_0$ (magnitude and direction).

(**b.**) (20 points)
Just after $t = 0$, six small masses ϵm, with $\epsilon \ll 1$, are added at the six *other* corners. They are added "at speed" so that, just after $t = 0$, $\vec{\omega}$ is unaffected. Describe the subsequent motion of the angular momentum $\vec{L}$ as seen in the body system.

2. (35 points)
A small particle with initial kinetic energy T_0 moves in the (two-dimensional) xy plane. It is incident on a circular region $\sqrt{x^2 + y^2} < R$ within which its potential energy is increased by a constant U_0, where $0 < U_0 < T_0$.

(**a.**) (20 points)
For this part, you are not given the impact parameter of the particle, but you are told that it *barely penetrates* the circular region before leaving it. In terms of T_0, U_0, and/or R, calculate the scattering angle Θ of the particle (where $\Theta \equiv 0$ for no scattering; $\Theta \equiv \pi$ for backscattering).

(**b.**) (15 points)
For this part, you are given no information about how far the particle penetrates into the cylindrical region. But, with respect to the origin, you *are* told that the particle's impact parameter b ranges uniformly over all possible values. In analogy to the three-dimensional *total scattering cross section* σ, a two-dimensional *total scattering length* ℓ may be defined as

$$\ell \equiv \int_{-\pi}^{\pi} \left| \frac{\mathrm{d}b}{\mathrm{d}\Theta} \right| \mathrm{d}\Theta \; .$$

In terms of T_0, U_0, and/or R, find ℓ for this part.

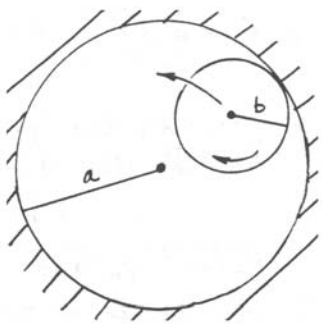
3. (35 points)
A particle of mass m moves in a potential well

$$U(x, y) = \tfrac{1}{2}k_1(x - y)^2 + \tfrac{1}{2}k_2(x + y)^2 \; .$$

(**a.**) (15 points)
Find the system's natural frequencies of oscillation.

(**b.**) (10 points)
Find the normal coordinates (within a multiplicative constant).

(**c.**) (10 points)
If $k_1 = k_2$, find a cyclic coordinate q such that $\partial\mathcal{L}/\partial q = 0$. (*Note that q need not be the same as x or y.*)



4. (30 points)
A hoop of radius b, mass m rolls without slipping within a fixed cylinder of inside radius a under the influence of gravity g. A line drawn from the center of the cylinder to the center of the hoop makes an angle ϕ with the vertical.

(**a.**) (15 points)
Find the period of small oscillations about equilibrium $\phi = 0$.

(**b.**) (15 points)
Find the period of large oscillations when the amplitude is $|\phi|_{\max} = \pi/2$. Express your answer as a definite integral containing only quantities that are given.

5. (30 points)
Three thin steel rods of length l and mass m are welded into a rigid equilateral triangle that, in equilibrium, lies in a horizontal plane. Each corner is attached to a horizontal tabletop by a spring of constant k. The corners can move only in the vertical direction, and their amplitude of motion is $\ll l$.

Calculate the normal frequencies of oscillation.

[*Hint: Choose as generalized coordinates the (actual minus equilibrium) lengths of the springs. The easiest way to solve the secular equation is to guess one of the normal modes and its frequency.*]

6. (40 points)
A string with mass per unit length μ and tension τ is stretched between two supports at $x = 0$ and $x = L$. At $t = 0$ the string satisfies the initial conditions

$$y(x,0) = 0$$
$$\frac{\partial y}{\partial t}(x,0) = K \exp\left(-(x - \tfrac{L}{2})^2 / 2\sigma^2\right) ,$$

where $\sigma \ll L$.

(**a.**) (15 points)
Calculate $y(x, t_1)$, where $t_1 \equiv L\sqrt{\mu/\tau}$.

(**b.**) (15 points)
Calculate $\frac{\partial y}{\partial t}(x, t_1)$.

(**c.**) (10 points)
Assuming that the string is tuned to middle C ($\nu = 256$ Hz), give the lowest frequency $\nu_{\min}$ that characterizes the sound of *this* waveform.