University of California, Berkeley
Physics 105 Fall 2006 (*Strovink*)

EXAMINATION 2

Directions: Do all 3 problems, which have unequal weight. This is an 80-minute closed-book closed-note exam except for *Short Course in Classical Mechanics*, Taylor, a copy of anything posted on the course web site, and anything in your own handwriting (not a Xerox of someone else's writing). Laptops, palmtops, calculators, and cellphones should be turned off. Use a bluebook. Do not use scratch paper – otherwise you risk losing part credit. Show all your work. Cross out rather than erase any work that you wish the grader to ignore. Justify what you do. Express your answer in terms of the quantities specified in the problem. Box or circle your answer.

Problem 1. (35 points)
A rocket orbits the sun elliptically, far from any planet. The rocket's controllers plan to destroy it by crashing it into the sun. To do this, they fire the rocket's engine for a very brief time, exerting a fixed force $\mathbf{F}$ during that time.

(**a.**) (15 points)
To minimize the time Δt during which the rocket must be fired, where in the orbit, and in what direction, should $\mathbf{F}$ occur? Why?

(**b.**) 20 points)
Suppose instead that the rocket's orbit is circular. Its controllers fire the rocket for a minimum time Δt_e so that it barely escapes the solar system. Calculate the ratio

$$\mathcal{R} = \frac{\Delta t_e}{\Delta t_c} ,$$

where Δt_c is the firing time that would have been required to crash the same rocket into the sun.

Problem 2. (30 points)
A simple plane pendulum consists of a mass m attached to a string of length h. After the pendulum is set into motion, the length of the string is shortened at a constant rate

$$\frac{\mathrm{d}h}{\mathrm{d}t} = -\alpha = \text{constant.}$$

The suspension point remains fixed. Choose as generalized coordinate the angle θ made by the string with respect to the vertical.

Calculate the Hamiltonian $\mathcal{H}$ for this system. Express it in terms of θ and the canonical momentum p_θ that is conjugate to θ. Is $\mathcal{H}$ equal to the total energy? Is $\mathcal{H}$ conserved?

Problem 3. (35 points)
A thin disklike turntable of radius R and mass m rotates frictionlessly about its symmetry axis. Initially two men of combined mass m stand on the edge of the turntable at opposite ends of a diameter. After the turntable (and men) are set into rotation with angular velocity ω, the turntable-men system is subjected to no further external torques.

(**a.**) (20 points)
As seen in a reference frame attached to the turntable, both men walk toward the turntable's axis with the same velocity v_0, where $v_0 \ll \omega R$. Their journey requires them to exert a continuous effort. At what radius r ($0 \le r \le R$) is their effort greatest?

(**b.**) (15 points)
Instead of walking toward the axis, one man falls off the turntable while the other remains standing. Put the origin at the turntable's center with the z axis along the axis of rotation, and approximate the remaining man as a point mass at height h above the x axis. What is the inertia tensor I of the turntable-remaining man system?