University of California, Berkeley
Physics 105 Fall 2006 (*Strovink*)

EXAMINATION 1

Directions: Do all 3 problems, which have unequal weight. This is an 80-minute closed-book closed-note exam except for *Short Course in Classical Mechanics*, Taylor, a copy of anything posted on the course web site, and anything in your own handwriting (not a Xerox of someone else's writing). Laptops, palmtops, calculators, and cellphones should be turned off. Use a bluebook. Do not use scratch paper – otherwise you risk losing part credit. Show all your work. Cross out rather than erase any work that you wish the grader to ignore. Justify what you do. Express your answer in terms of the quantities specified in the problem. Box or circle your answer.

Problem 1. (25 points)
A *passive* transformation E consists, in the following order, of
• A coordinate system reflection $(x_1, x_2, x_3) \to (-x_1, x_2, x_3)$.
• A counterclockwise rotation of the coordinate system by $\pi/2$ about $\hat{\mathbf{e}}_3$.
• A coordinate system reflection $(x_1, x_2, x_3) \to (x_1, x_2, -x_3)$.

(**a.**) (15 points)
If $\tilde{x}' = E\tilde{x}$, where $\tilde{x}'$ and $\tilde{x}$ are 3×1 matrix representations of the same vector in the transformed and untransformed coordinate systems, respectively, write the 3×3 matrix E.

(**b.**) (10 points)
Could the rotation represented by E be built up out of an infinite sequence of normal (not parity-inverting) infinitesimal rotations? Explain.

Problem 2. (35 points)
During $-\infty < t < 0$, a linear oscillator satisfying the equation of motion

$$\ddot{x} + \omega_0\dot{x} + \omega_0^2 x = \frac{F_x(t)}{m}$$

is driven at its resonant frequency by a force per unit mass

$$\frac{F_x(t)}{m} = a_0 \cos\omega_0 t \ ,$$

where a_0 and ω_0 are positive real constants.

(**a.**) (20 points)
Find $x(0)$ and $\dot{x}(0)$ at $t = 0$.

(**b.**) (15 points)
At $t = 0$ the driving force is turned off. Find $x(t)$ for $t > 0$.

Problem 3. (40 points)
On a horizontal frictionless table, positions may be specified by Cartesian coordinates (x, y) or circular coordinates (ρ, θ), where as usual $\rho \equiv \sqrt{x^2 + y^2}$ and $\theta \equiv \arctan y/x$. The table supports a very small puck of mass m; at the origin, a small hole is drilled through the table. By means of a frictionless massless string that is threaded through the hole, the puck is attached to a massless Hooke's law spring underneath the table. The length of the string is such that the spring is relaxed when the puck is at the origin. Therefore, away from the origin, the puck has potential energy

$$U = \tfrac{1}{2}k\rho^2 \ ,$$

where k is the spring constant.

(**a.**) (10 points)
Write down the Lagrangian *in circular coordinates*.

(**b.**) (10 points)
Find two constants of the puck's motion. Express them in terms of $\rho, \dot{\rho}, \theta, \dot{\theta}$, and constants.

(**c.**) (10 points)
Use the Euler-Lagrange equation to obtain the second-order differential equation of motion for the puck's radius ρ. Substitute a constant of the motion to eliminate $\dot{\theta}$ from this equation.

(**d.**) (10 points)
Under the initial conditions $x(0) = x_0 \neq 0$, $\dot{x}(0) = \dot{y}(0) = y(0) = 0$, solve for the motion of the puck.